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NAVAL POSTGRADUATE SCHOOL

Monterey, California



THESIS

THE EFFECT OF SEARCH RADIUS ON SEARCH EFFICIENCY
WHEN THE TARGETS MUST BE VISITED

by

Charles Edward Laidlaw

March 1977

Thesis Advisor:

A.R. Washburn

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THE EFFECT OF SEARCH RADIUS ON SEARCH EFFICIENCY WHEN THE
TARGETS MUST BE VISITED

by

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Submitted in partial fulfillment of the
requirements for the degree of

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from the
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March 1977

ABSTRACT

This paper proposes and evaluates a general search function which can be used for determining the efficiency of a search in which the searcher is required to visit the maximum number of targets in an allotted period of time. A lower bound on this function is presented as a consequence of the "semimyopic" strategy utilized in the simulation.

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I. INTRODUCTION

Two of the factors that must be considered in planning the search of a large area are the search radius (maximum detection range) and search speed. Due to a decrease in the maximum detection range of some sensors with increasing search speed, the interaction of these two factors and their ultimate effect on search efficiency must be accounted for. The development of some analytical means by which the planner could account for the effect of changing search radius on search efficiency prompted this research.

The concept investigated was that of a single searcher required to visit as many targets as possible, in an infinite field of uniformly distributed targets (poisson field), in an allotted period of time. The searcher was placed randomly in the field of targets at the start of the search. Visiting the same target more than once was not allowed. Search speed was assumed to be much greater than target speed and, hence, all targets were considered stationary in the field.

The search strategy employed becomes extremely important as the search radius increases with respect to expected target spacing. It seems logical to always go to the nearest detected, but unvisited, target; however, such a strategy allows the searcher to backtrack and travel over previously covered areas. It may well be better for the searcher to go to a fresh area.

Figures 2 through 7 show the typical track of a searcher employing a "go to the nearest target" or "semimyopic"

strategy for increasing values of search radius. In each case the searcher started from the center of a poisson field of 1000 targets. If no targets were within the search radius then a search course was taken away from the center of mass of all previously visited targets, until a target was detected.

II. BACKGROUND

The objective of the searcher is to maximize the number of targets visited in a unit of time, subject to the following assumptions:

- Search field is of infinite size.
- The targets are uniformly distributed over the field.
- The targets are stationary.

This search problem can be divided into essentially three distinct situations or cases.

A. CASE I - SEARCH RADIUS MUCH LESS THAN EXPECTED TARGET SPACING

In the limit this is the classical random search problem described in Ref. 1. If the search radius (r) is very small, then the number of targets visited is approximately equal to the number of targets detected.

B. CASE II - SEARCH RADIUS MUCH GREATER THAN EXPECTED TARGET SPACING

In the limit this approaches a situation of totally unrestricted visibility, and the searcher can pick and choose his optimum track.

C. CASE III - ANY OTHER SEARCH RADIUS

This is by far the most interesting case and the objective implies finding an optimal search strategy when anything more sophisticated than random search can be employed. Possible search strategies could include any one of, or a combination of, the following:

- Random search.
- Search along a steady course.
- Patterned search.
- Go to the nearest target if one or more targets held.
- Go to the nearest area of "high target density" if more than one held.

III. ANALYSIS AND THEORY

If the number of targets visited is fixed, then the average total distance traveled (l) to visit all of the targets is a function of the number of targets visited (n), the search radius (r), and the target density (p). Thus,

$$l = l(r, p, n)$$

A. DIMENSIONAL ANALYSIS

By dimensional analysis,

$$l\sqrt{p} = f(r\sqrt{p}, n)$$

since all distances can be measured in terms of $1/\sqrt{p}$. Assuming that $l(r, p, n)$ is asymptotically linear in n , then

$$\lim_{n \rightarrow \infty} 2f(r\sqrt{p}, n)/n = 1/g(r\sqrt{p})$$

The primary objective of this thesis was the discovery of this dimensionless function, $g(x)$, where $x = r\sqrt{p}$.

B. EXPECTATIONS FOR $g(x)$

Let,

$$D_i = \text{Distance From } (i-1)^{\text{st}} \text{ To } i^{\text{th}} \text{ Target}$$

Where $i = 1, 2, 3, \dots$

Then,

$$\begin{aligned} P(D_1 \geq d) &= P(\text{No Tgts in } \pi d^2) \\ &= 1 - e^{-\pi d^2 p} \end{aligned}$$

and,

$$E(D_1) = \int_0^{\infty} e^{-\pi p y^2} dy = 1/(2\sqrt{p}) \quad p > 0$$

Since the first detection is based on the searcher being placed in a virgin field, it follows that the probability of being close to an unvisited target is greater at the start than at any other time of the search. Thus, intuitively,

$$E(D_1) \leq E(D_i) \quad i \geq 2$$

But,

$$1 = E(D_1 + \dots + D_n)$$

so,

$$\begin{aligned} 1/g(x) &= \lim_{n \rightarrow \infty} 2E(D_1 + \dots + D_n)\sqrt{p}/n \\ &= \lim_{n \rightarrow \infty} 2 \sum_{i=1}^n E(D_i)\sqrt{p}/n \\ &\geq \lim_{n \rightarrow \infty} 2nE(D_1)\sqrt{p}/n = 2E(D_1)\sqrt{p} = 1 \end{aligned}$$

Thus,

$$g(x) \leq 1$$

If x is small, then one can conclude that classical random search, as described by Koopman [Ref. 1], applies and that the number of targets visited is approximately equal to the number of targets detected. Thus, applying the standard

"cookie cutter" model, gives

$$N = ap = 2rlp = 2l\sqrt{p}(r\sqrt{p})$$

Where a = Area Searched

and for small x ,

$$g(x) = x$$

C. AN ESTIMATOR FOR $g(x)$

Let,

$$Z = 2(D_1 + \dots + D_n)\sqrt{p}/n$$

then,

$$\begin{aligned} E(Z) &= 2E(D_1 + \dots + D_n)\sqrt{p}/n \\ &= 2f(r\sqrt{p}, n)/n \\ &\approx 1/g(x) \quad \text{if } n \text{ is large} \end{aligned}$$

Thus, for large n , Z is an estimator for $1/g(x)$.

This thesis used $1/Z$ as an estimator for $g(x)$ even though $E(1/Z) \geq g(x)$. Because the $\text{Var}(Z)$ was small when n was large,

$$E(1/Z) \approx 1/E(Z) \approx g(x)$$

The actual estimator utilized was,

$$g(x) = N/(2L\sqrt{p})$$

Where N = Number Of Targets Visited

L = Total Distance To n^{th} Target

IV. METHODOLOGY

The development of optimal search strategies for the full spectrum of search radii to be employed was dismissed from the outset, and two practical search strategies settled on. The resulting strategy may best be defined as being a semimyopic strategy. This semimyopic strategy provided a lower bound on $g(x)$, and hence on the expected number of targets that can be visited by a searcher.

A. DIRECT MOVE STRATEGY

If there are targets within the search radius of the searcher, he visits the nearest target.

B. INDIRECT MOVE STRATEGY

If no targets are held initially, the search is conducted along a randomly determined course. If, having visited at least two targets, there are no targets within the search radius of the searcher, the search course is determined by the position of the last target visited and the center of mass of all previously visited targets. In both cases, the search course is always directed towards areas not previously searched and the searcher always visits the first target detected.

V. RESULTS

Figure 1 shows the plot of $g(x)$ vs x and its one-sigma boundaries obtained by computer simulation.

Assuming that a search is to be conducted in a poisson field of targets, the planner can, by using the relationship developed earlier and Figure 1, determine the expected number of targets that can be visited by passing close aboard. Furthermore, by knowing the effects that search speed has on the search radius of the sensor to be employed, he can compare expected search results and select the most efficient or favorable speed at which to conduct the search.

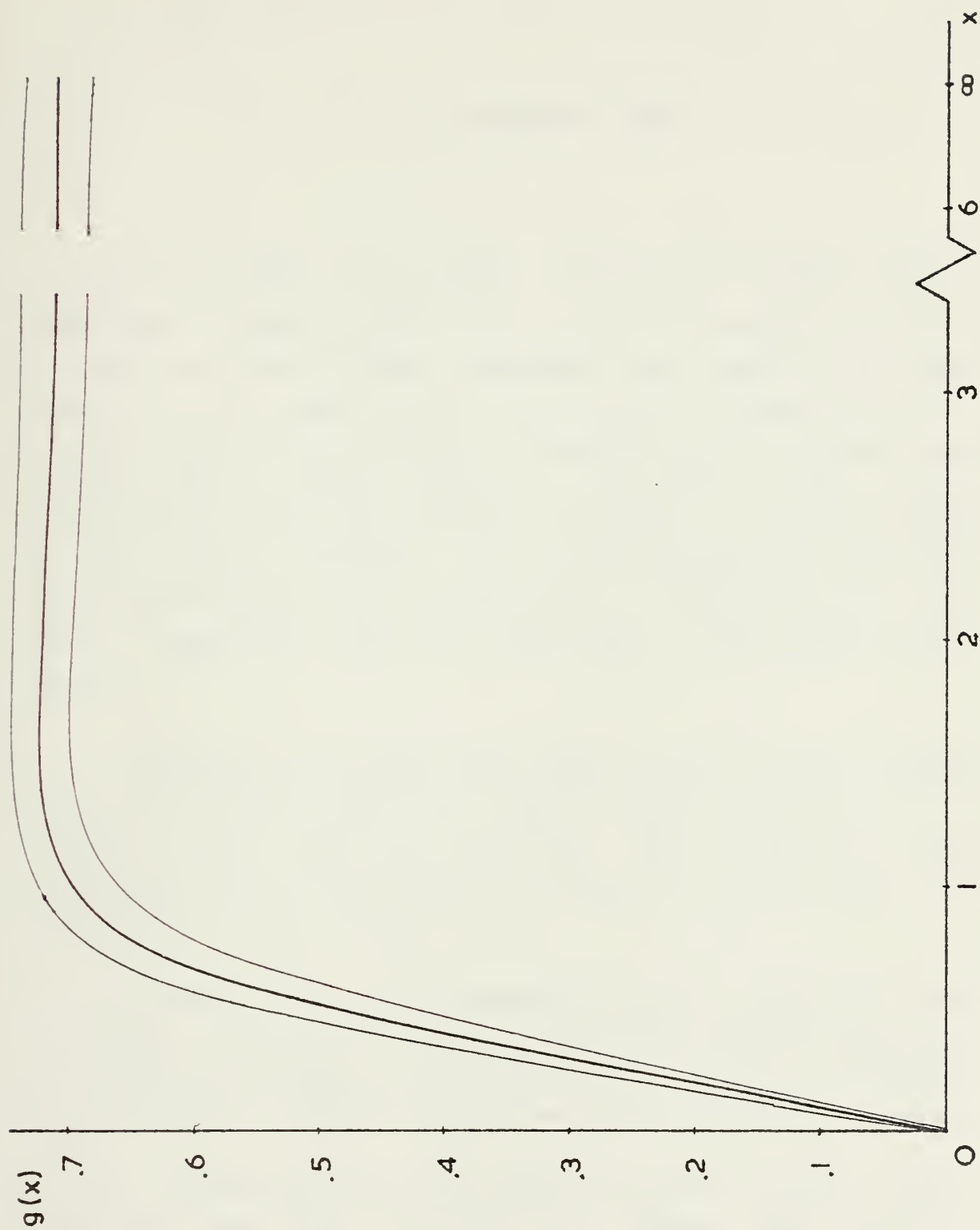


Figure 1 - $g(x)$ vs x

VI. SIMULATION MODEL

A unit square with 10,000 uniformly distributed random targets was used to represent an infinite field, and a simulation search program was written in FORTRAN and run on the IBM 360. The searcher commenced his search from the center of the unit square and, after a series of direct (target within search radius of searcher) and indirect (no target within search radius of searcher) moves, would terminate when (1) the searcher was within one search radius of the fields boundary, or (2) the distance to the next target was greater than the distance to the boundary of the field. Appendix A contains a flow chart of the models basic logic.

A minimum of 40 runs were made through the same random fields for various values of search radius, equivalent to x ranging from 0.025 to infinity. The results of the simulation runs are contained in Appendix B, and Figure 1 is the resulting plot of $g(x)$ vs x .

The effect of x , and consequently search radius, on the searchers track and the number of targets visited are shown in figures 2 through 7. The 1000 target field is identical in each case. "Direct" refers to the Direct Move Strategy.

The results from all runs made in 10,000 target fields indicated that for $x \geq 1.0$ the Direct Move Strategy prevailed, and that for $x \geq 4.0$ the searcher always held the next target.

The simulation required 170K on the IBM 360 and the time required for a run through 40 poisson fields of 10,000 targets each varied from about one minute, for small values of search radius, up to about fifteen minutes, for large values of search radius.

VII. CONCLUSION

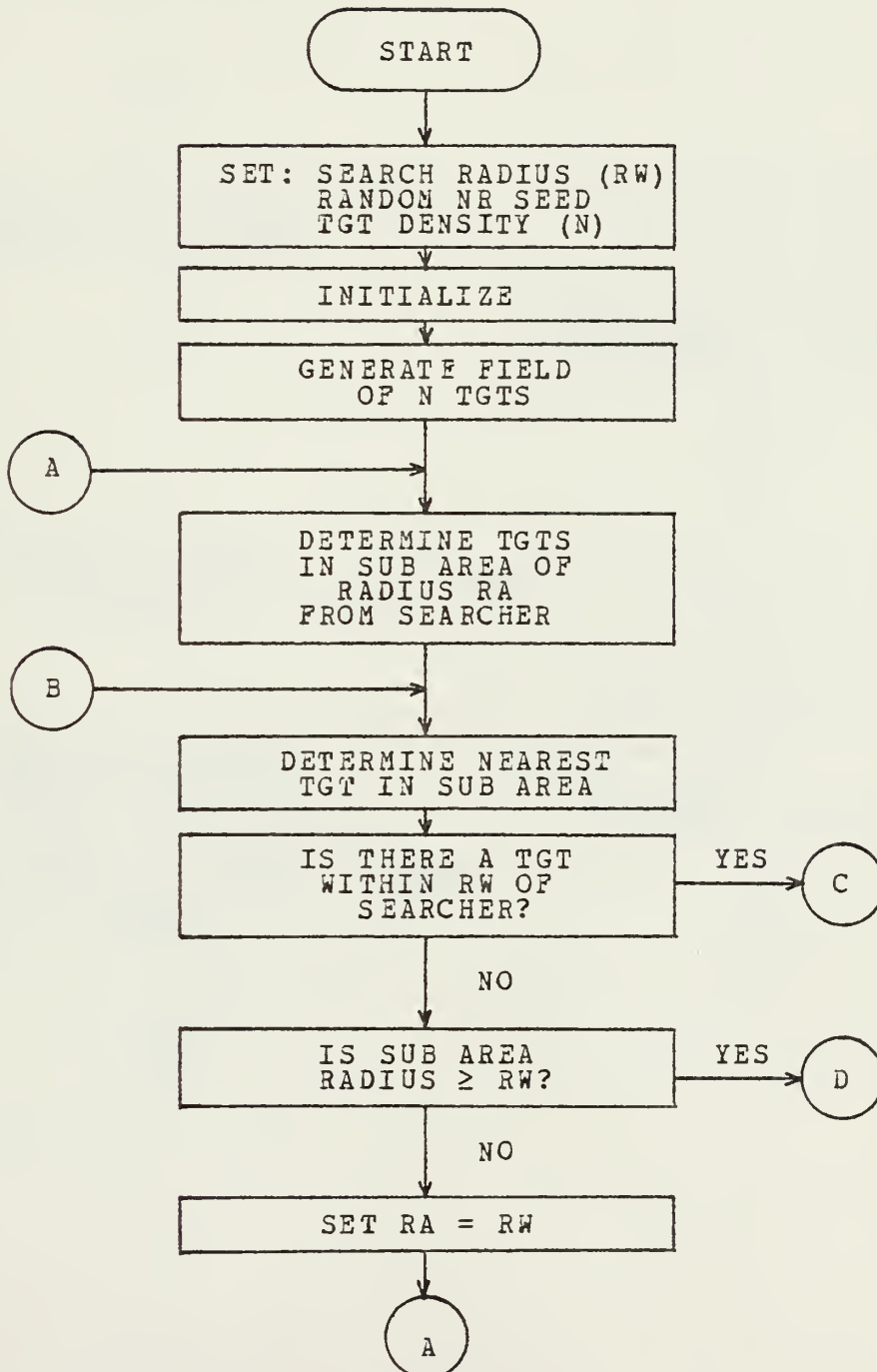
The results obtained for small values of x correlate nicely with random search theory. Based on random search theory, it was expected that $g(x)$ would equal x for small values of x , and Figure 1 shows that this relationship holds for values of x up to approximately 0.5.

It was anticipated that $g(x)$ would approach some fixed value with increasing x , yet Figure 1 and the data obtained indicate a slight peak at about $x = 2.0$. The only plausible explanation for this peak is that the searcher is penalized for employing a myopic strategy in a hypermetropic situation.

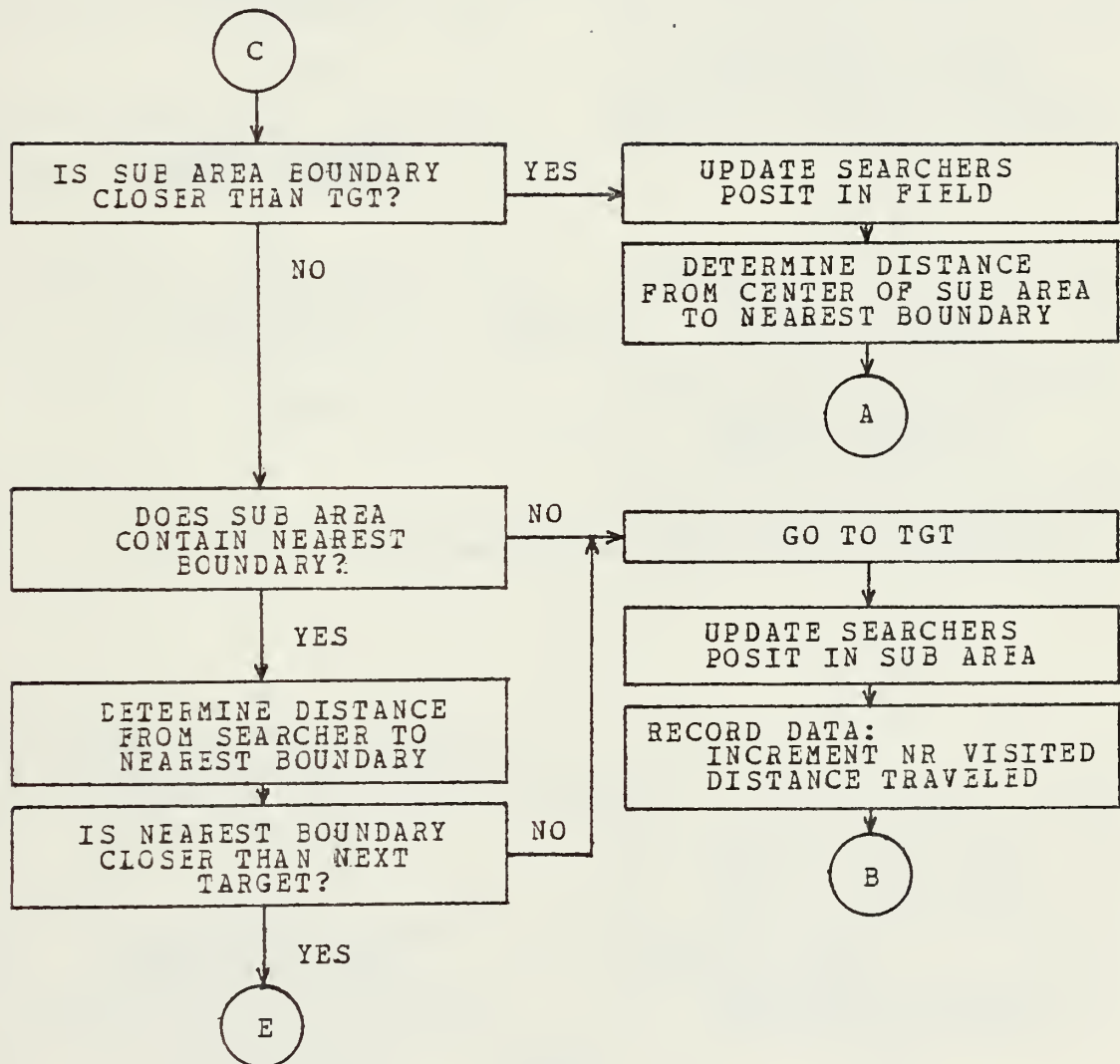
A close inspection of Figures 2 through 7 leads one to conclude that a strategy of going to the nearest area of high target density could substantially increase the number of targets visited per unit distance traveled, and subsequently $g(x)$. To verify this, seven visual trials were made by three individuals using the same 1000 target field used in Figures 2 through 7. The objective of the individuals was to visit as many targets as possible with unrestricted visibility. The resulting values of $g(x)$ ranged from 0.7622 to 0.8401 with $\bar{g}(x) = 0.8075$. By computer simulation, for unrestricted visibility, $g(x) = 0.6720652$ for this field. These results tend to substantiate the belief that a searcher employing a myopic strategy in a hypermetropic situation is penalized, and the earlier claim that the semimyopic strategy served to provide a lower bound on $g(x)$.

APPENDIX A

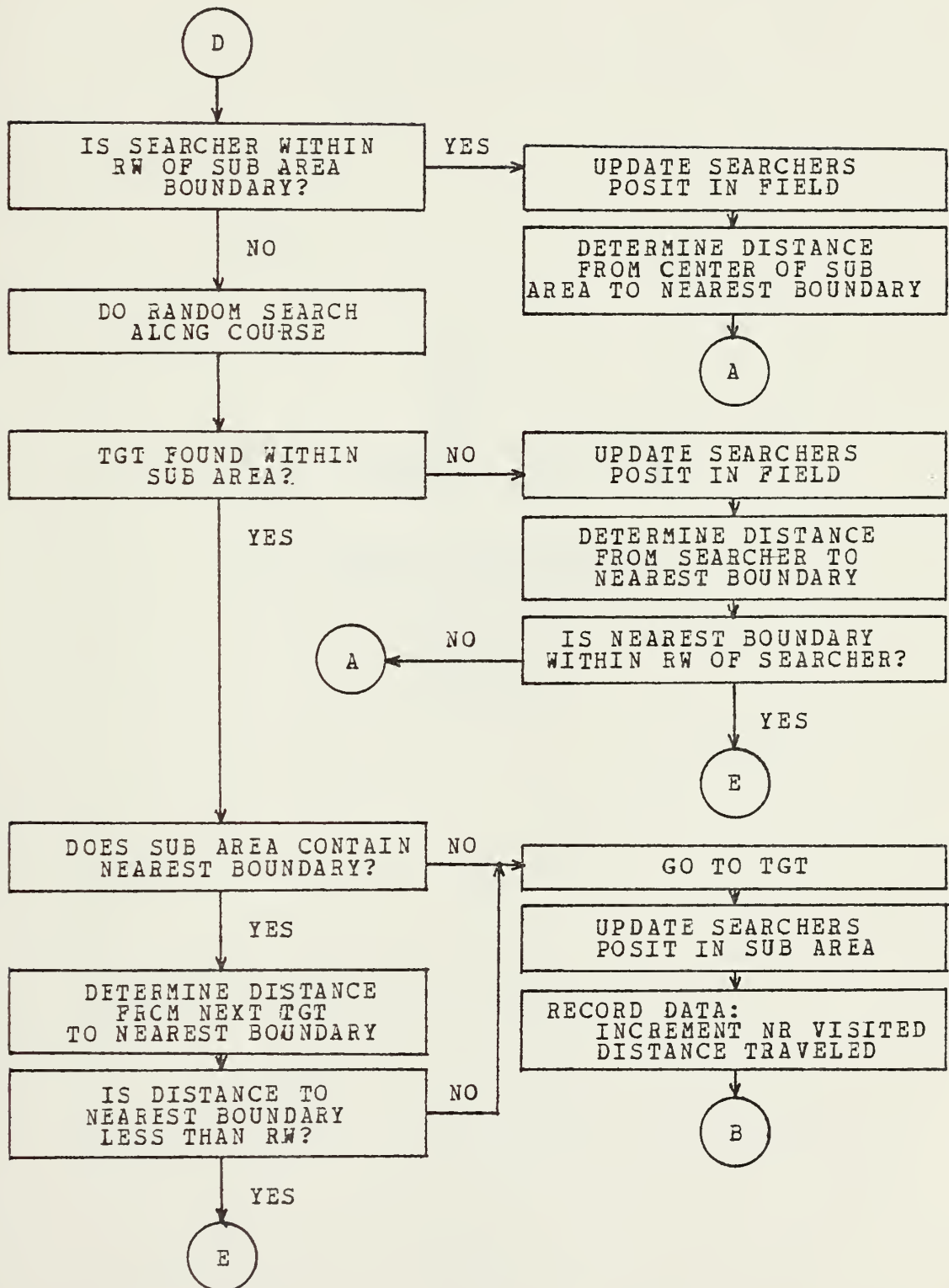
FLOW CHART FOR SIMULATION MODEL

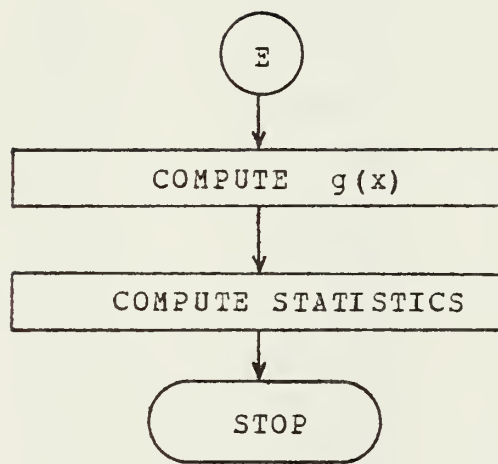


(DIRECT MOVE)



(INDIRECT MOVE)



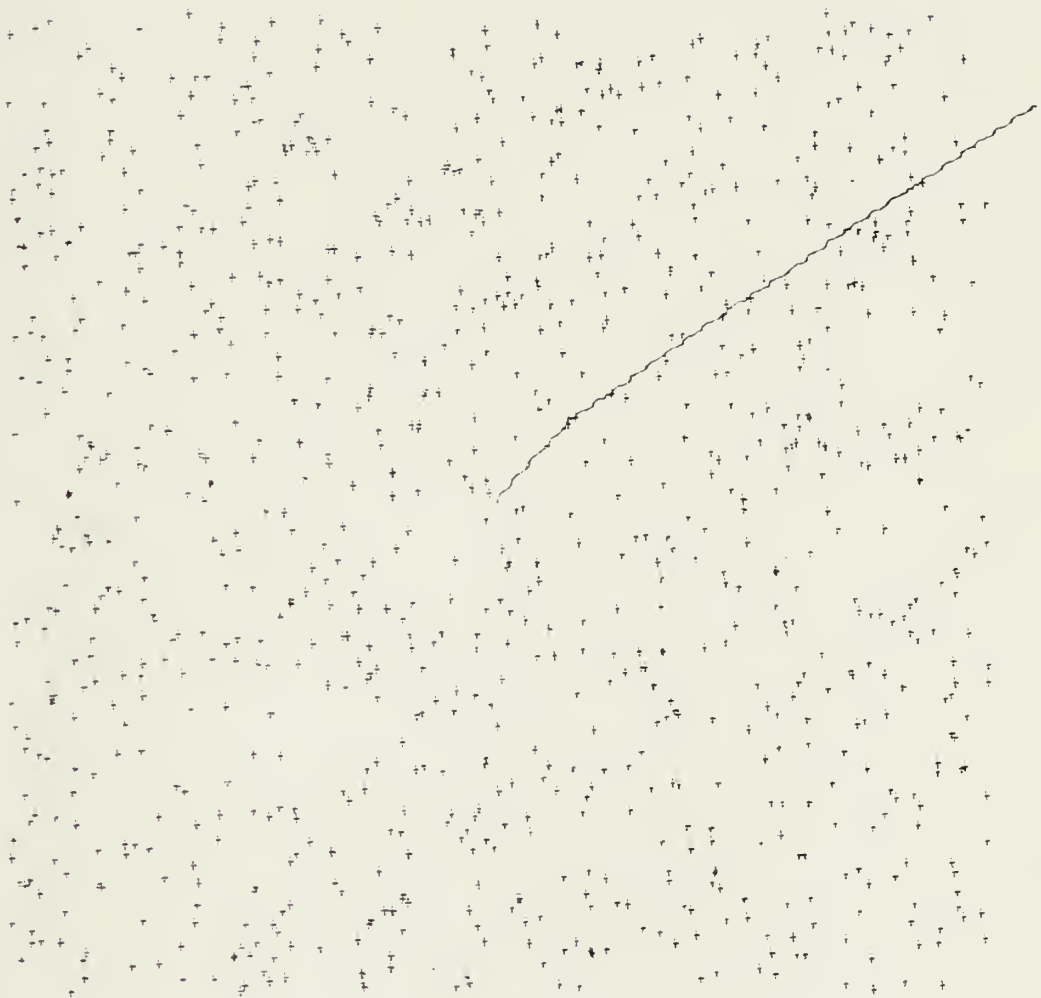


APPENDIX B

COMPUTER OUTPUT

x	Number Of Runs	g (x)	Sigma
0.025	40	0.0424551	0.0315156
0.05	40	0.0678127	0.0360265
0.075	40	0.0753077	0.0260168
0.1	40	0.1006204	0.0282282
0.125	80	0.1318160	0.0342952
0.25	80	0.2574581	0.0476352
0.5	80	0.4808525	0.0567625
0.75	80	0.6374704	0.0501638
0.875	80	0.6741247	0.0418967
1.0	80	0.6959368	0.0319426
1.125	80	0.7058284	0.0312344
1.25	80	0.7143928	0.0279759
1.375	80	0.7196197	0.0244312
1.5	80	0.7194164	0.0238991
1.625	80	0.7176019	0.0225489
2.0	40	0.7181079	0.0200270
2.5	40	0.7136417	0.0247005
3.0	40	0.7161400	0.0227902
4.5	40	0.7124111	0.0252671
6.0	40	0.7110950	0.0260190
8.0	40	0.7114341	0.0255649
INF	80	0.7117410	0.0231861

TABLE I

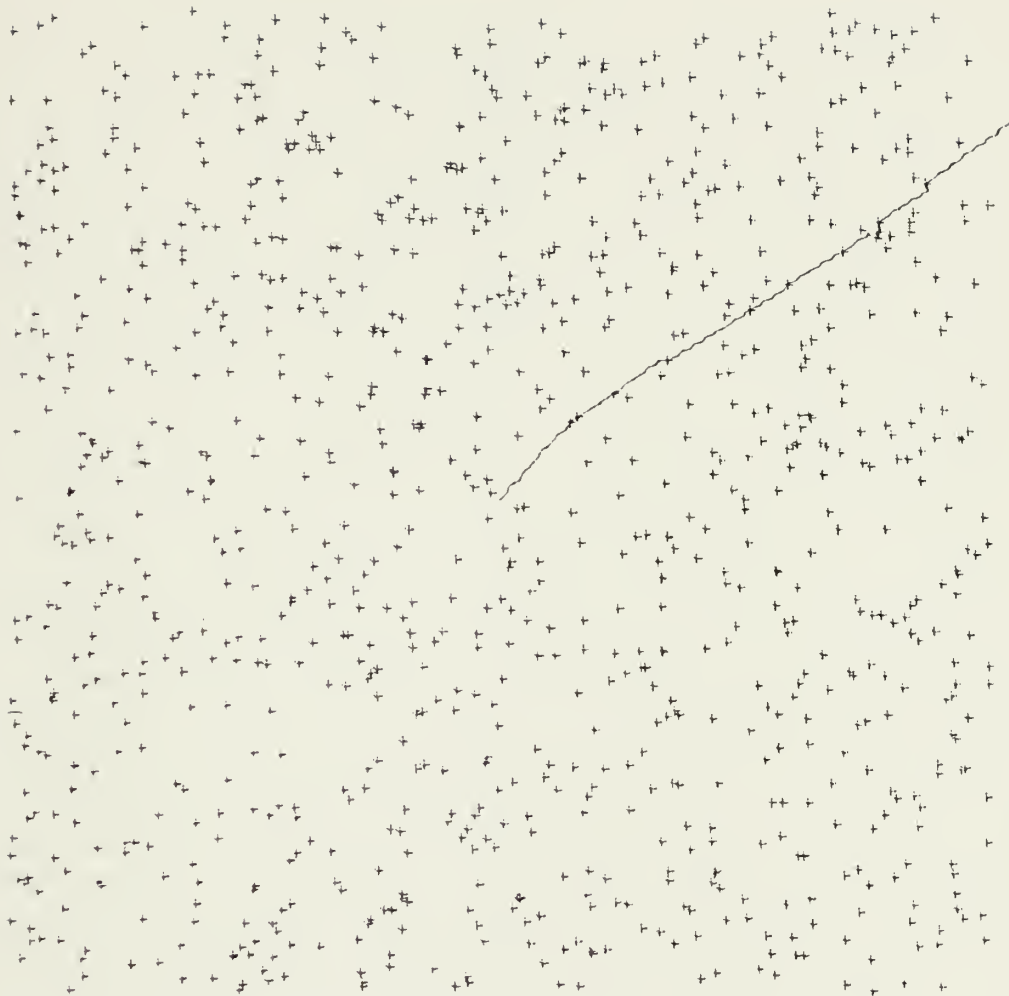


Field of 1000 Targets

Search Radius = 0.01 Inch

6 Targets Visited, NONE Of Which Were "Direct"

Figure 2 - TRACK WITH $x = 0.125$



Field of 1000 Targets

Search Radius = 0.02 Inch

11 Targets Visited, 1 Of Which Was "Direct"

Figure 3 - TRACK WITH $x = 0.25$

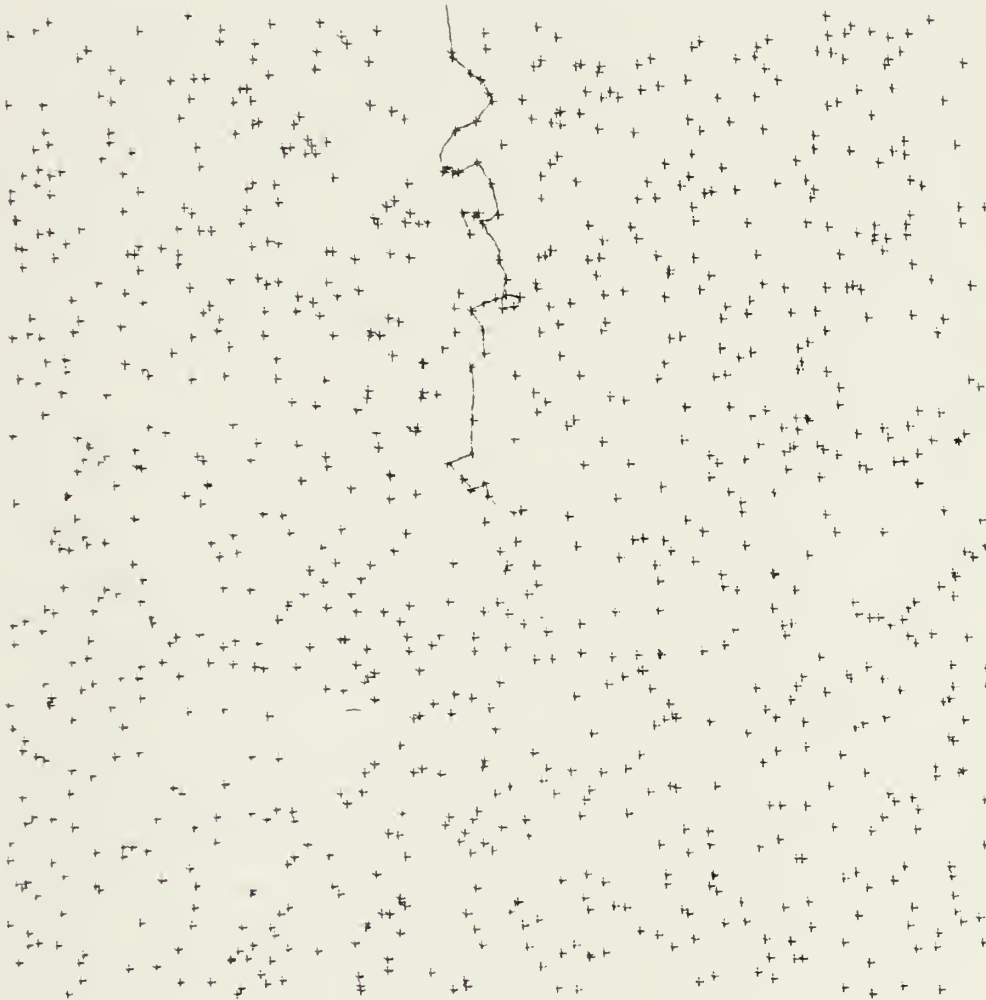


Field of 1000 Targets

Search Radius = 0.04 Inch

24 Targets Visited, 10 Of Which Were "Direct"

Figure 4 - TRACK WITH $x = 0.50$



Field of 1000 Targets

Search Radius = 0.08 Inch

39 Targets Visited, 33 Of Which Were "Direct"

Figure 5 - TRACK WITH $x = 1.0$



Field of 1000 Targets

Search Radius = 0.16 Inch

80 Targets Visited, ALL Of Which Were "Direct"

Figure 6 - TRACK WITH $x = 2.0$



Field of 1000 Targets

Search Radius = 0.32 Inch

80 Targets Visited, ALL Of Which Were "Direct"

Figure 7 - TRACK WITH $x = 4.0$

LIST OF REFERENCES

1. Operations Evaluation Group, Office of the Chief of Naval Operations, Report 56, Search and Screening, by B.S. Kocpman, p.. 28, 1946.

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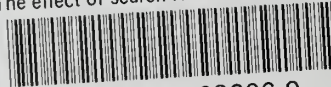
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